

# Cost-Benefit Analysis of Two Similar Warm Standby Systems Subject to Failure Due to Very High Magnitude Earthquake and Heavy Rain Causing Deadliest Flood

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**Abstract**—In this paper we have taken failure due to very high magnitude earthquake, and heavy rain causing deadliest flood. When the main unit fails then warm standby system becomes operative. Failure due to heavy rain causing deadliest flood cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

## 1. INTRODUCTION

### Nepal Very high magnitude earthquake Death Toll Passes more than 7,000

A magnitude 7.8 very high magnitude earthquake is devastating enough, but with the hypocenter a mere 15 kilometers (9.3 miles) below the surface, that shaking is brutally intense in a localized area. Along with damage from intense, local shaking, this very high magnitude earthquake has likely triggered countless landslides and destabilized even more slopes, increasing the risk of more landslides during the upcoming monsoon season.

### List of deadliest floods

Below is a list of the deadliest floods worldwide, showing events with death tolls at or above 100,000 individuals

Death toll	Event	Location	Date
2,500,000–3,700,000	1931 China floods	China	1931
900,000–2,000,000	1887 Yellow River (Huang He) flood	China	1887
500,000–700,000	1938 Yellow River (Huang He) flood	China	1938

231,000	Banqiao Dam failure, result of Typhoon Nina. Approximately 86,000 people died from flooding and another 145,000 died during subsequent disease.	China	1975
230,000	Indian Ocean tsunami	Indonesia	2004
145,000	1935 Yangtze river flood	China	1935
100,000+	St. Felix's Flood, storm surge	Netherlands	1530
100,000	Hanoi and Red River Delta flood	North Vietnam	1971
100,000	1911 Yangtze river flood	China	1911

In this paper we have taken failure due to very high magnitude earthquake, and heavy rain causing deadliest flood. When the main operative unit fails then warm standby system becomes operative. Failure due to heavy rain causing deadliest flood cannot occur simultaneously in both the units. After failure the unit undergoes repair facility of Type- I or Type- II by ordinary repairman, Type III or Type IV by multispecialty repairman immediately when failure due to very high magnitude earthquake and heavy rain causing deadliest flood. The repair is done on the basis of first fail first repaired.

## 2. ASSUMPTIONS

- $\lambda_1, \lambda_2, \lambda_3$  are constant failure rates when failure due to very high magnitude earthquake, failure due to heavy rain causing deadliest flood respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are  $G_1(t), G_2(t)$  and  $G_3(t), G_4(t)$ .
- The failure due to heavy rain causing deadliest flood is non-instantaneous and it cannot come simultaneously in both the units.

3. The repair starts immediately after failure due to very high magnitude earthquake and failure due to heavy rain causing deadliest flood and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

### Symbols for states of the System

#### Superscripts O, WS, HMEF, HRFF,

Operative, Warm Standby, failure due to very high magnitude earthquake, failure due to heavy rain causing deadliest flood respectively

#### Subscripts nhmef, nhmef, hrff, ur, wr, uR

No failure due to very high magnitude earthquake, failure due to very high magnitude earthquake, failure due to heavy rain causing deadliest flood, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 3, 10; Down states – 4, 5, 6, 7, 8,9,11, regeneration point – 0, 1, 2, 3, 8, 9, 10

#### States of the System

**0**( $O_{nhmef}$ ,  $WS_{nhmef}$ ) One unit is operative and the other unit is warm standby and there is no failure due to very high magnitude earthquake of both the units.

**1**( $HMEF_{hmef, urI}$ ,  $O_{nhmef}$ ) The operating unit failure due to very high magnitude earthquake is under repair immediately of Type- I and standby unit starts operating with no failure due to very high magnitude earthquake

**2**( $HRFF_{hrff, urII}$ ,  $O_{nhmef}$ ) The operative unit failure due to heavy rain causing deadliest flood and undergoes repair of Type II and the standby unit becomes operative with no failure due to very high magnitude earthquake

**3**( $HRFF_{hrff, urIII}$ ,  $O_{nhmef}$ ) The first unit failure due to heavy rain causing deadliest flood and under Type-III multispecialty repairman and the other unit is operative with no failure due to very high magnitude earthquake

**4**( $HMEF_{hmef, uR1}$ ,  $HMEF_{hmef, wrI}$ ) The unit failed due to HMEF resulting from failure due to very high magnitude earthquake under repair of Type- I continued from state 1 and the other unit failed due to HMEF resulting from failure due to very high magnitude earthquake is waiting for repair of Type-I.

**5**( $HMEF_{hmef, uR1}$ ,  $HRFF_{hrff, wrII}$ ) The unit failed due to HMEF resulting from failure due to very high magnitude earthquake is under repair of Type- I continued from state 1 and the other unit fails due to heavy rain causing deadliest flood is waiting for repair of Type- II.

**6**( $HRFF_{hrff, uRII}$ ,  $HMEF_{hmef, wrI}$ ) The operative unit failed due to heavy rain causing deadliest flood is under repair continues from state 2 of Type –II and the other unit failed due to HMEF resulting from failure due to very high magnitude earthquake is waiting under repair of Type-I.

**7**( $HRFF_{hrff, uRII}$ ,  $HMEF_{hmef, wrII}$ ) The one unit failed due to heavy rain causing deadliest flood is continued to be under repair of Type II and the other unit failed due to HMEF resulting from failure due to very high magnitude earthquake is waiting for repair of Type-II.

**8**( $HMEF_{hmef, urIII}$ ,  $HRFF_{hrff, wrII}$ ) The one unit failure due to very high magnitude earthquake is under multispecialty repair of Type-III and the other unit failed due to heavy rain causing deadliest flood is waiting for repair of Type-II.

**9**( $HMEF_{hmef, urIII}$ ,  $HRFF_{hrff, wrI}$ ) The one unit failure due to very high magnitude earthquake is under multispecialty repair of Type-III and the other unit failed due to heavy rain causing deadliest flood is waiting for repair of Type-I

**10**( $O_{nhmef}$ ,  $HRFF_{hrff, urIV}$ ) The one unit is operative with no failure due to very high magnitude earthquake and warm standby unit fails due to heavy rain causing deadliest flood and undergoes repair of type IV.

**11**( $O_{nhmef}$ ,  $HRFF_{hrff, uRIV}$ ) The one unit is operative with no failure due to very high magnitude earthquake and warm standby unit fails due to heavy rain causing deadliest flood and repair of type IV continues from state 10.

#### Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$$\begin{aligned}
 p_{01} &= \lambda_1 / (\lambda_1 + \lambda_2 + \lambda_3), p_{02} = \lambda_2 / (\lambda_1 + \lambda_2 + \lambda_3), \\
 p_{0,10} &= \lambda_3 / (\lambda_1 + \lambda_2 + \lambda_3), p_{10} = pG_1^*(\lambda_1) + qG_2^*(\lambda_2), \\
 p_{14} &= p - pG_1^*(\lambda_1) = p_{11}^{(4)}, p_{15} = q - qG_1^*(\lambda_2) = p_{12}^{(5)}, \\
 p_{23} &= pG_2^*(\lambda_1) + qG_2^*(\lambda_2), p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)}, \\
 p_{27} &= q - qG_2^*(\lambda_2) = p_{28}^{(7)}, p_{30} = p_{82} = p_{91} = 1, \\
 p_{0,10} &= pG_4^*(\lambda_1) + qG_4^*(\lambda_2), \\
 p_{10,1} &= p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}, \\
 p_{10,2} &= q - qG_4^*(\lambda_2) = p_{10,2}^{(11)} \tag{1}
 \end{aligned}$$

We can easily verify that

$$p_{01} + p_{02} + p_{03} = 1, p_{10} + p_{14} (=p_{11}^{(4)}) + p_{15} (=p_{12}^{(5)}) = 1,$$

$$p_{23} + p_{26} (=p_{29}^{(6)}) + p_{27} (=p_{28}^{(7)}) = 1, p_{30} = p_{82} = p_{91} = 1$$

$$p_{10,0} + p_{10,1}^{(11)} (=p_{10,1}) + p_{10,2}^{(12)} (=p_{10,2}) = 1 \tag{2}$$

And mean sojourn time is

$$\mu_0 = E(T) = \int_0^{\infty} P[T > t] dt$$

**3. MEAN TIME TO SYSTEM FAILURE**

$$\begin{aligned} \emptyset_0(t) &= Q_{01}(t)[s] \emptyset_1(t) + Q_{02}(t)[s] \emptyset_2(t) + Q_{0,10}(t)[s] \emptyset_{10}(t) \\ \emptyset_1(t) &= Q_{10}(t)[s] \emptyset_0(t) + Q_{14}(t) + Q_{15}(t) \\ \emptyset_2(t) &= Q_{23}(t)[s] \emptyset_3(t) + Q_{26}(t) + Q_{27}(t), \emptyset_3(t) = Q_{30}(t)[s] \emptyset_0(t) \\ \emptyset_{10}(t) &= Q_{10,0}(t)[s] \emptyset_{10}(t) + Q_{10,1}(t)[s] \emptyset_1(t) + Q_{10,2}(t)[s] \emptyset_2(t) \end{aligned} \tag{3-6}$$

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-6) and solving for

$$\emptyset_0^*(s) = N_1(s) / D_1(s) \tag{7}$$

where

$$\begin{aligned} N_1(s) &= \{Q_{01}^* + Q_{0,10}^* Q_{10,1}^*\} [Q_{14}^*(s) + Q_{15}^*(s)] + \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^*\} [Q_{26}^*(s) + Q_{27}^*(s)] \\ D_1(s) &= 1 - \{Q_{01}^* + Q_{0,10}^* Q_{10,1}^*\} Q_{10}^* - \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^*\} Q_{23}^* Q_{30}^* - Q_{0,10}^* Q_{10,0}^* \end{aligned}$$

Making use of relations (1) & (2) it can be shown that

$\emptyset_0^*(0) = 1$ , which implies that  $\emptyset_0(t)$  is a proper distribution.

$$\begin{aligned} \text{MTSF} = E[T] &= \frac{d}{ds} \emptyset_0^*(s) \Big|_{s=0} \\ &= (D_1'(0) - N_1'(0)) / D_1(0) \\ &= (\mu_0 + \mu_1 (p_{01} + p_{0,10} p_{10,1}) + (p_{02} + p_{0,10} p_{10,2})(\mu_2 + \mu_3) + \mu_{10} p_{0,10} / (1 - (p_{01} + p_{0,10} p_{10,1}) p_{10} - (p_{02} + p_{0,10} p_{10,2}) p_{23}) - p_{0,10} p_{10,0}) \end{aligned}$$

where

$$\begin{aligned} \mu_0 &= \mu_{01} + \mu_{02} + \mu_{0,10}, \mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}, \\ \mu_2 &= \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}, \mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2} \end{aligned}$$

**4. AVAILABILITY ANALYSIS**

Let  $M_i(t)$  be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$$\begin{aligned} M_0(t) &= e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t}, M_1(t) = p G_1(t) e^{-\lambda_1 t} \\ M_2(t) &= q G_2(t) e^{-\lambda_2 t}, M_3(t) = G_3(t), M_{10}(t) = G_4(t) e^{-\lambda_3 t} \end{aligned}$$

The point wise availability  $A_i(t)$  have the following recursive relations

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t) \\ A_1(t) &= M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t), \\ A_2(t) &= M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]A_8(t) + q_{29}^{(6)}(t)[c]A_9(t) \\ A_3(t) &= M_3(t) + q_{30}(t)[c]A_0(t), \\ A_8(t) &= q_{82}(t)[c]A_2(t), A_9(t) = q_{91}(t)[c]A_1(t), \\ A_{10}(t) &= M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(12)}(t)[c]A_2(t) \end{aligned} \tag{8-15}$$

Taking Laplace Transform of eq. (8-15) and solving for

$$\begin{aligned} \tilde{A}_0(s) \\ \tilde{A}_0(s) &= N_2(s) / D_2(s) \end{aligned} \tag{16}$$

where

$$\begin{aligned} N_2(s) &= \{ \hat{q}_{0,10} \bar{M}_{10} + \bar{M}_0 \} [ \{ 1 - \hat{q}_{11}^{(4)} \} \{ 1 - \hat{q}_{28}^{(7)} \hat{q}_{82} \} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} ] + \{ \hat{q}_{01} + \hat{q}_{0,10} \hat{q}_{10,1}^{(11)} \} [ \bar{M}_1 \{ 1 - \hat{q}_{28}^{(7)} \hat{q}_{82} \} + \hat{q}_{12}^{(5)} \hat{q}_{23} \bar{M}_3 + \bar{M}_2 ] + \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}^{(12)} \} [ \hat{q}_{23} \bar{M}_3 \{ 1 - \hat{q}_{11}^{(4)} \} + \hat{q}_{29}^{(6)} \hat{q}_{91} \bar{M}_1 ] \\ D_2(s) &= \{ 1 - \hat{q}_{11}^{(4)} \} \{ 1 - \hat{q}_{28}^{(7)} \hat{q}_{82} \} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} - \{ \hat{q}_{01} + \hat{q}_{0,10} \hat{q}_{10,1}^{(11)} \} [ \hat{q}_{10} \{ 1 - \hat{q}_{28}^{(7)} \hat{q}_{82} \} + \hat{q}_{12}^{(5)} \hat{q}_{23} \hat{q}_{30} ] - \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}^{(12)} \} [ \hat{q}_{23} \hat{q}_{30} \{ 1 - \hat{q}_{11}^{(4)} \} + \hat{q}_{29}^{(6)} \hat{q}_{91} \hat{q}_{10} ] \end{aligned}$$

(Omitting the arguments s for brevity)

The steady state availability

$$\begin{aligned} A_0 &= \lim_{t \rightarrow \infty} [A_0(t)] = \lim_{s \rightarrow 0} [s \tilde{A}_0(s)] = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)} \end{aligned}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s \rightarrow 0} \frac{N_2(s) + s N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \tag{17}$$

The expected up time of the system in (0,t] is

$$\begin{aligned} \lambda_u(t) &= \int_0^t A_0(x) dx \text{ So that } \bar{\lambda}_u(s) = \frac{\tilde{A}_0(s)}{s} = \frac{N_2(s)}{s D_2(s)} \end{aligned} \tag{18}$$

The expected down time of the system in (0,t] is

$$\lambda_{\bar{d}}(t) = t - \lambda_{u}(t) \text{ So that}$$

$$\bar{\lambda}_{\bar{d}}(s) = \frac{1}{s^2} - \bar{\lambda}_{u}(s) \quad (19)$$

Similarly, we can find out

1. The expected busy period of the server when there is failure due to very high magnitude earthquake, and heavy rain causing deadliest flood in (0,t]-R<sub>0</sub>
2. The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H<sub>0</sub>
3. The expected number of visits by the multispecialty repairman Type-III or Type-IV for repairing the identical units in (0,t]-W<sub>0</sub>, Y<sub>0</sub>.

**5. BENEFIT-FUNCTION**

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to very high magnitude earthquake, and heavy rain causing deadliest flood, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t] is

$$C = \lim_{t \rightarrow \infty} (C(t)/t) = \lim_{s \rightarrow 0} (s^2 C(s)) = K_1 A_0 - K_2 R_0 - K_3 H_0 - K_4 W_0 - K_5 Y_0$$

where

K<sub>1</sub> - revenue per unit up-time, K<sub>2</sub> - cost per unit time for which the system is busy under repairing, K<sub>3</sub> - cost per visit by the repairman type- I or type- II for units repair,

K<sub>4</sub> - cost per visit by the multispecialty repairman Type- III for units repair,

K<sub>5</sub> - cost per visit by the multispecialty repairman Type- IV for units repair

**6. CONCLUSION**

After studying the system, we have analyzed graphically that when the failure rate due to very high magnitude earthquake

and due to heavy rain causing deadliest flood increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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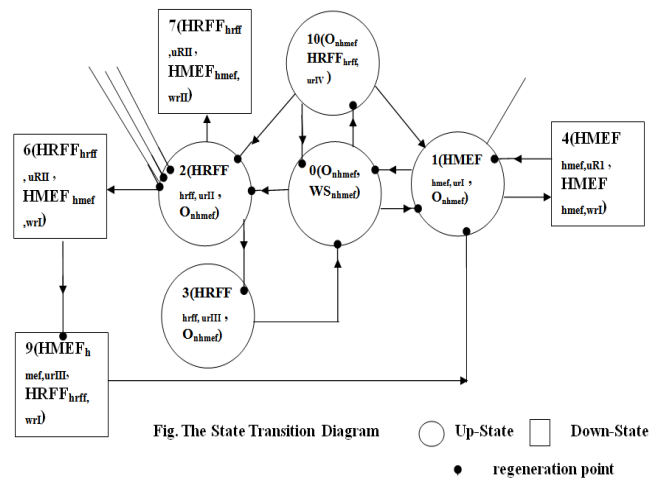


Fig. The State Transition Diagram ○ Up-State □ Down-State ● regeneration point